Received: *April 18, 2001*, Accepted: *November 15, 2001*
Revised: *November 15, 2001*

The polarization of M5-branes and little string theories with reduced supersymmetry

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Abstract: We construct an M-theory dual of a 6-dimensional self-dual string theory with reduced supersymmetry, along the lines of Polchinski and Strassler. We find that upon perturbing the $(2,0)$ theory with an R-current, the M5-branes polarize into a wrapped Kaluza Klein monopole, whose isometry direction is along the R current. We investigate the properties of this theory.

Keywords: M-Theory, AdS/CFT Correspondance.

JHEP11(2001)032

Contents

1. Introduction

A while ago Polchinski and Strassler [1] found a string theory dual of a 4-dimensional SYM theory with reduced supersymmetry. They perturbed the $\mathcal{N} = 4$ theory by giving mass to the hypermultiplets. In the dual bulk theory this corresponded to turning on transverse RR and NS 3-form field strengths which polarized [2] the D3-branes into D5 or NS5-branes. This extension of the original AdS/CFT duality [3] had implications in both directions. It allowed for a detailed string theory interpretation of domain walls, baryon vertices, confinement, etc. and it illustrated a way in which string theory can cure some singularities in supergravity.

Since then, papers have appeared generalizing the PS construction to theories in other dimensions (M2-branes [4], D2-branes [5, 6]), as well as to 4-dimensional theories on orbifolds [7], with temperature [8], even less susy [9], etc. [10].

One of the possible extensions of [1] left unexplored has been to the $AdS_7 \times S^4$ geometry. In the same line of thought from [1, 4, 5] one would expect the dual of a little string theory with reduced supersymmetry to be given in terms of an M theory bulk containing polarized M5-branes. It is the purpose of this paper to explore this possibility.

Since the degrees of freedom of a large number of M5-branes are not known, giving a matrix description to this polarization (like in the $D_p \rightarrow D_{p+2}$-brane polarization) is not possible. The only way to describe this polarization is thus to find the action of the object in which the M5-branes are polarized, and to show that there exists a configuration with non-zero radius which is energetically stable.

In order to understand which operators one needs to turn on to obtain polarized M5-branes, and find what they might polarize into, it is worth thinking about the polarization of a large number of D4-branes into D6-branes. These configurations exist and are understood very well (they are T-dual to the D3 $\rightarrow$ D5 system discussed in [1]). They are produced by turning on a fermion scalar bilinear on the worldvolume of the D4-branes, which corresponds to a combination of RR 1-form and NSNS 2-form on the transverse space.[1]

One expects upon lifting this polarized configuration to M theory to obtain a configuration in which M5-branes polarize into Kaluza-Klein monopoles (KKM). One of the directions of the M5-brane is special in that it is along the isometry direction of the monopole. The fermion scalar bilinear on the D4-brane can come from the dimensional reduction of a fermion scalar or a fermion vector bilinear oriented in the M-theory direction. The bulk RR 1-form and NSNS 2-form come from a graviton with one leg on the 11'th direction (the graviphoton) and, respectively, from a 3-form potential with one leg on the 11'th direction. Since these bulk fields transform as vectors under the Lorentz $SO(5,1)$ group, we expect them to correspond to a fermion bilinear which is a vector oriented in the 11'th direction. This same polarization configuration can be obtained by T-dualizing the D3 $\rightarrow$ NS5 polarization of [1] and obtaining a D4-brane polarized into a IIA KK monopole. The D4 $\rightarrow$ IIA KKM configuration lifts to the same configuration as above, with the M-theory direction identified differently.

One might wonder whether there are other possible polarizations for M5-branes. It appears possible to polarize it into a codimension 4 higher object, namely an M9-brane. However, since we are interested in M-theory configurations only, and the M9 is a brane solution of *massive* 11d supergravity (sugra), we shall not explore this possibility.

Nevertheless, it does not appear possible to polarize the M5's into a KKM which does not have its isometry direction along the brane, simply because the original M5-branes do not have that isometry.

[1] We should note, however, that the naive expectation that the D4-D6 polarization configuration is still supersymmetric fails.

There are however a few problems with which an astute reader might already be concerned. The first one has to do with the tension of the KK monopole. As we shall discuss, the tension of the KKM is proportional to the square of the length of the 11'th dimension. Thus, as we open up the 11'th dimension the monopole becomes more and more massive. If for example in IIA we had N D4-branes polarizing into one D6-brane, as we open the 11'the dimension the ratio of the charges changes, and at one point the KKM in which the M5-branes polarize will be heavier than the 5-branes, in which case it will not be treatable as a probe. We will address these problems in due time.

We will be thus attempting to describe a $(2,0)$ theory perturbed with an R-current, using the AdS/CFT bulk dual of this theory. A $(2,0)$ theory perturbed with a similar R-current has been found to appear on M5-branes which are pinned to the center of a perturbed Taub-NUT space [11]. However, in the regime considered in [11] the KKM charge dominates, and the brane on which the perturbed $(2,0)$ theory lives is just a probe. In our case the branes where the perturbed $(2,0)$ theory lives are the source of the geometry, and the KKM charge appears as a perturbation. It would be interesting to explore possible interpolations between the two regimes.

While this paper was being completed, [12] appeared, in which the polarization of M5-branes into KKM's was also suggested.

2. Perturbing the $AdS_7 \times S_4$ background

2.1 The sugra set-up

The 11-dimensional supergravity bosonic lagrangian is

$$\mathcal{L} = R * 1 - \frac{1}{2} F_4 \wedge * F_4 + \frac{1}{6} F_4 \wedge F_4 \wedge C_3 \,, \tag{2.1}$$

where $F_4 = dC_3$, and $*$ is the 11-dimensional Hodge star. One can define the 6-form potential C_6 as the Poincaré dual of C_3

$$F_7 \equiv dC_6 = *F_4 + \frac{1}{2} C_3 \wedge F_4 \,. \tag{2.2}$$

The M5-brane is a soliton solution of 11d sugra

$$\left(ds^2\right)^{\mathbf{0}} = Z^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu + Z^{-1/3} dx_{11}^2 + Z^{2/3} dx^i dx^j \delta_{ij} \,, \tag{2.3}$$

$$F_7^{\mathbf{0}} = \partial_i Z^{-1} dx^0 \wedge \cdots \wedge dx^4 \wedge dx^{11} \wedge dx^i \,, \tag{2.4}$$

where the brane is aligned along $\mu = 0,1,2,3,4,11$, the transverse coordinates are $i = 5, \ldots, 9$, and Z is the harmonic function on the transverse space.

It is well known that the background geometry of a large number of coinciding M5-branes is $AdS_7 \times S_4$, and that 11d sugra compactified on $AdS_7 \times S_4$ gives a dual description to the 6d $(2,0)$ CFT theory living on the worldvolume of the M5-branes.

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For N coincident M5-branes the harmonic function is

$$Z = \frac{R^3}{r^3}\,, \qquad r^2 = x^i x^i\,, \qquad R^3 \equiv \pi N l_p^3\,. \tag{2.5}$$

The $AdS_7 \times S_4$ geometry becomes transparent if we redefine $r = 4R/z_0^2$. The AdS_7 radius is thus twice the S_4 radius. For later need, we give the Poincaré dual of (2.4)

$$F_4^{\mathbf{0}} = \frac{1}{4!}\epsilon_{ijklm}\partial_m Z dx^i \wedge dx^j \wedge dx^k \wedge dx^l\,, \tag{2.6}$$

where ϵ_{ijklm} is the flat antisymmetric tensor restricted to the transverse coordinates.

We want to perturb this background with a non-normalizable mode of the field which is dual to a fermion mass operator in the CFT. The fermions of $(2,0)$ CFT transform in the $\underline{4}$ spinor representation of the R-symmetry group SO(5). Thus a fermion bilinear which is a $\underline{10}$ of SO(5) corresponds to a 2-form (or by Poincaré duality a 3-form) field strength perturbation in the space transverse to the branes. Since 11d SUGRA has only a 4-form field strength and a graviton, the only way to obtain a transverse 3-form is to consider a 4-form with a leg along one of the longitudinal directions. Similarly the transverse 2-form is the field strength of the graviphoton.

We can see that only a fermion bilinear which is a Lorentz vector is a chiral primary. Thus, to turn on a fermion bilinear which is chiral we need to break Lorentz covariance and introduce a preferred direction, or equivalently, a Killing vector. We shall see later that this breaking of Lorentz covariance is the key to the construction in which the M5-branes polarize into Kaluza-Klein monopoles.

The background geometry admits a Killing vector

$$D^{\mathbf{0}}_{(\Lambda}k_{\Pi)} = 0, \qquad \Lambda = 0, \ldots, 9, 11 \tag{2.7}$$

with $k_\Lambda = Z^{-1/3}\delta_{\Lambda 11}$. We note that the contravariant Killing vector is simply $k^\Lambda = \delta^{\Lambda 11}$. From now on we will use $^{\mathbf{0}}$ to denote background fields and $^{\mathbf{1}}$ to denote perturbed fields.

We want to turn on the non-normalizable modes of the perturbations $\delta g_{i11} \equiv g^{\mathbf{1}}_{i11} \equiv h_{11i}$ and $\delta C^{\mathbf{0}}_{11ij} \equiv C^{\mathbf{1}}_{11ij}$ which only depend on the transverse coordinates. The metric perturbation is related to the 3-form perturbation by

$$\delta R_{11i} = \delta\left\{\frac{-1}{12}\left(F_{11\ldots}F_i^{\;\cdots} - \frac{1}{12}g_{11i}F_{\ldots}F^{\cdots}\right)\right\}. \tag{2.8}$$

The variation of the Ricci tensor

$$\delta R_{\Lambda\Pi} = \frac{1}{2}\left(h^{\Omega}_{\;\Lambda;\Pi\Omega} + h^{\Omega}_{\;\Pi;\Lambda\Omega} - h_{;\Lambda\Pi} - \Box h_{\Lambda\Pi}\right) \tag{2.9}$$

in the background (2.3) is

$$\delta R_{11i} = \frac{1}{2}\partial_j\left(Z^{-1}\partial_j\left(h_{11i}Z^{1/3}\right) - Z^{-1}\partial_i\left(h_{11j}Z^{1/3}\right)\right) - \frac{1}{24}h_{11i}\partial_m Z\partial_m Z Z^{-8/3}\,. \tag{2.10}$$

Combining this with (2.8) we obtain

$$d\left[Z^{-1}\left(*_5 d\left(H_1 Z^{1/3}\right)+G_3\right)\right]=0\,, \tag{2.11}$$

where $*_5$ denotes the flat Hodge dual in the transverse space, $G_3 \equiv (1/3!)F^{\mathbf{1}}_{11ijk}dx^i \wedge dx^j \wedge dx^k$, and $H_1 \equiv h_{11i}dx^i$.

Similarly, from the first order variation of the F_4 field equation:

$$d *^{\mathbf{0}} F_4^{\mathbf{1}} + d *^{\mathbf{1}} F_4^{\mathbf{0}} = 0 \tag{2.12}$$

we can obtain after a few straightforward steps:

$$d\left[Z^{-1}\left(d\left(H_1 Z^{1/3}\right)+*_5 G_3\right)\right]=0\,. \tag{2.13}$$

In order to find the exact form of the bulk 2- and 3-forms corresponding to the fermion bilinear we need to relate them to transverse tensors, and to solve (2.11) and (2.13).

2.2 Fermion bilinears and transverse tensors

The fermion R-current can be related to a tensor on the transverse space by analyzing their properties under the action of the R-symmetry group. We pair the 4 worldvolume fermions and 4 of the transverse space coordinates into complex variables

$$\begin{aligned} z_1 &= x^5 + ix^6\,, \qquad & z_2 &= x^8 + ix^9\,, \\ \Lambda_1 &= \lambda_1 + i\lambda_3\,, \qquad & \Lambda_2 &= \lambda_2 + i\lambda_4 \end{aligned} \tag{2.14}$$

and we notice that under an SO(5) rotation $z_i \to e^{i\phi^i} z_i$, the fermions transform as

$$\begin{aligned} \Lambda_1 &\longrightarrow e^{i(\phi^1-\phi^2)/2}\Lambda_1\,, \\ \Lambda_2 &\longrightarrow e^{i(\phi^1+\phi^2)/2}\Lambda_2\,. \end{aligned} \tag{2.15}$$

Thus, a diagonal fermion "mass" term behaves in the same way under SO(5) rotations as

$$T_2 = -\,\mathrm{Re}[m_1 dz_1 \wedge d\bar{z}_2 + m_2 dz_1 \wedge dz_2]\,. \tag{2.16}$$

To make T_2 a Lorentz vector on the worldvolume all we have to do is tensor it with $\wedge dx^{11}$. We can also define on the transverse space the Hodge dual to T_2

$$T_3 = *_5 T_2 = \mathrm{Re}[m_1 dz_1 \wedge d\bar{z}_2 + m_2 dz_1 \wedge dz_2] \wedge dx^7\,. \tag{2.17}$$

Note that if we set either fermion mass to zero, there is a remaining global U(1) symmetry.

2.3 The first order perturbation

A general bulk 3-tensor with the SO(5) transformation properties of a fermion bilinear will be a combination of T_3 and V_3, multiplied by a power of r, where V_3 is given in the appendix. Thus, we can express the bulk 3-tensor H_3 and 2-tensor $F_2 \equiv d(H_1 Z^{1/3})$ as

$$\begin{aligned} F_2 &= \left(\frac{r}{R}\right)^p (aV_2 + bT_2)\,, \\ G_3 &= \left(\frac{r}{R}\right)^p (AV_3 + BT_3)\,, \end{aligned} \tag{2.18}$$

where T_2 and T_3 were found above, and V_2, V_3 together with some useful identities are given in the appendix. Using the Bianchi identities for the field strengths F and G we get the constraints

$$pb = 2a\,, \qquad pB = 3A\,. \tag{2.19}$$

Substituting (2.18) and (2.19) into the harmonic form eq. (2.11) and (2.13) we obtain the system:

$$\begin{aligned} (p+3)\left(b + \frac{pb}{2} + B\right) - 3\left(\frac{pB}{3} - \frac{pb}{2}\right) &= 0\,, \\ (p+3)\left(b + B + \frac{pB}{3}\right) - 2\left(\frac{pB}{3} - \frac{pb}{2}\right) &= 0\,, \end{aligned} \tag{2.20}$$

which admits four real solutions: $(p = 0, b/B = -1), (p = -8, b/B = -1); (p = -5, b/B = 2/3), (p = -3, b/B = 2/3)$.

To interpret these modes, we need to know their conformal dimensions. To translate $G_3 = (1/3!)\partial_i C^{\mathbf{1}}_{11jk} dx^i \wedge dx^j \wedge dx^k$ to an inertial frame, we multiply (2.18) with $\sqrt{G^{11,11}G^{ii}G^{jj}G^{kk}} = Z^{-5/6} \propto r^{5/2}$.

Remembering that in AdS coordinates $z_0 \sim r^{-1/2}$, we find that the modes of G_3 corresponding to $p = -3$ and $p = -5$ behave near the boundary as z_0 and z_0^5, respectively. Thus, they are the non-normalizable and, respectively, normalizable solutions corresponding to an operator of conformal dimension $\Delta = 5$, which is our fermion bilinear. By matching the R-symmetry representation and conformal dimension with the table of CFT operators given in [13], we find that the CFT operator is indeed a Lorentz vector, bilinear in fermions — the R-symmetry current. In previous situations [1, 4, 5], the CFT operator corresponding to perturbations in the bulk matching the R-symmetry representation of a fermion bilinear was always a Lorentz scalar. In our case, the only bilinear in fermions which is a chiral primary operator is the R-current. The corresponding bulk perturbations are vectors under the SO(5, 1) Lorentz group, as anticipated. The non-normalizable mode $p = -3$ corresponds to the fermion "mass" perturbation, while the normalizable mode $p = -5$ corresponds to the vev $\langle\psi\psi\rangle$ [14].

We should note that the $p = -3$ perturbation is singular at the origin, and the polarization of the M5 into a KKM is the way M theory resolves this singularity. In a finite temperature set-up with the M5 background replaced by a thermal M5 black hole, the first order perturbation turns out to be the same as in the zero temperature case, but now the singularity is hidden behind the black hole horizon [15]. Thus an R-current corresponds to the following bulk perturbation:

$$Z^{-1}\left[*_5 d\left(H_1 Z^{1/3}\right) + G_3\right] = \xi T_3\,, \tag{2.21}$$

where ξ is a numerical constant which relates the physical mass of the fermions to the coefficient of the bulk mode. From now on we will absorb ξ in the fermion mass, and thus set it to 1 in the bulk.[2] We only wrote down the combination of modes which later will appear in the KKM lagrangian. It is interesting to notice that this particular combination is a harmonic function (as one can easily see from (2.11) and (2.13)). Therefore, it is determined by its value at the boundary (2.21), and does not depend on the particular form of Z.

Similarly, the modes with $p = -8$ and $p = 0$ correspond to a boundary operator of conformal dimension $\Delta = 11$. We can identify this operator to be $HH\psi\psi$, where H is the selfdual 3-form of the 6d $(2,0)$ tensor multiplet.

One might wonder if our perturbation (2.21) does not become stronger than the background is some regime. We will discuss this possibility in section 3.2.

3. Polarizing the M5-branes into KK monopoles

We would like to investigate whether a large number of M5-branes can polarize into KK monopoles. If the self-interacting potential of a KKM wrapped on a 2 sphere, with a very large M5 charge has a minimum when the 2-sphere radius is non zero, polarization occurs. In order to find this self interacting potential we first find the action for a test KKM with large M5 charge (n), in the potential created by a very large number $N \gg n$ of M5-branes. Anticipating, this test potential will be found to be independent of the positions of the N M5-branes. The full self interacting

[2]Let us briefly discuss how one can relate the fermion "mass" term of the theory living on the boundary of AdS_7 to the mass parameter of the bulk perturbations (2.16), (2.17) and (2.21). We start with the action of the M5-brane in an AdS_7 sugra superspace background (written for general backgrounds and manifest 6d general covariance by [16] and by [17] for flat sugra backgrounds and non-manifest 6d covariance; the complete expansion of the AdS_7 superfields can be found in [18]). After fixing the κ-symmetry, half of the θ superspace coordinates are eliminated, while the other half become fermions on the worldvolume. Thus, by specifying our perturbed AdS_7 background, we can search for terms bilinear in fermions. For instance, a Lorentz vector "mass" term comes from the WZ term (specifically from the pull-back of $C_6(X,\theta)$, where $E_M^{\underline{A}}$ is no longer zero by gauge choice). However, since the only parameter we introduced is the bulk mass (in units where the AdS radius is one), this implies that we have at most a numerical constant relating the boundary mass to the bulk mass.

potential can be found by bringing from infinity test KKM/M5 shells. Since the potential of these shells does not depend on the positions of the other M5-branes, the probe calculation with n replaced by N will give the full self-interacting potential.

3.1 The KKM action

As we explained in the introduction, a Kaluza-Klein monopole in 11d supergravity is an object which has 7 longitudinal directions, 3 transverse, as well as a special isometry direction. When one compactifies along this isometry direction one obtains a D6-brane in IIA. If we express the D6-brane tension in terms of the radius of the 11'th direction (R_{11}) and the 11'th-dimensional Planck length $l_{\rm P}$, we find that it is proportional to R_{11}^2:

$$T_{D6} = \frac{1}{g(2\pi)^6\alpha'^{7/2}} = \frac{R_{11}^2}{l_{\rm P}^9(2\pi)^6} = T_{KKM}\,.^3 \tag{3.1}$$

Therefore, the KKM cannot be thought of as some 7-brane wrapped on the isometry direction. This direction is special, and gets special treatment in the action.

The action for a Kaluza-Klein monopole (KKM) was given by [19, 20]. As expected, it depends on the background geometry Killing vector k^Λ corresponding to the isometry direction.

In the unperturbed $AdS_7 \times S_4$ background this Killing vector was found to be $k^{\Lambda\mathbf{0}} = \delta^{\Lambda 11}$. In the perturbed background, the variation $\delta k^\Lambda \equiv k^{\Lambda\mathbf{1}}$ can be found by solving

$$D^{\mathbf{0}}_{(\Lambda}k^{\mathbf{1}}_{\Pi)} = \frac{1}{2}(h_{11\Lambda;\Pi} + h_{11\Pi;\Lambda} - h_{\Lambda\Pi;11})\,. \tag{3.2}$$

It may not come as a surprise that k^Λ is still proportional with $\delta^{\Lambda 11}$, as one can easily check. However, since this Killing vector measures the length of the isometry direction at infinity, we normalize it as

$$k^\Lambda = \frac{R_{11}}{l_{\rm P}}\delta^{\Lambda 11}\,. \tag{3.3}$$

The KKM action is the sum of a Born-Infeld (BI) and a Wess-Zumino (WZ) piece. The BI part (in an 11-dimensional sugra background) reads[4]

$$S_{\rm BI} = -\frac{1}{(2\pi)^6 l_{\rm P}^7}\int d^7\xi k^2\sqrt{|\det(D_aX^\Lambda D_bX^\Pi g_{\Lambda\Pi} + (2\pi\alpha')k^{-1}\mathcal{F}_{ab})|}\,, \tag{3.4}$$

where the covariant derivative is defined with respect to a gauge field $D_aX^\Lambda = \partial_aX^\Lambda + A_ak^\Lambda$ which is not an independent field: $A_a = k^{-2}\partial_aX^\Lambda k_\Lambda$. The field strength

[3]Using $g = l_{\rm P}^{-3/2}R_{11}^{3/2}$ and $\alpha' = l_{\rm P}^3R_{11}^{-1}$.

[4]Identifying the reduced KKM action with the D6 action, not only allows us to relate their tensions, but the BI actions as well. As a consequence, the factor $2\pi\alpha'$ appears in the KKM action, with the understanding that it is a parameter which depends on $l_{\rm P}$ and R_{11} as specified in (3.1).

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of the 1-form field $b = b_a d\xi^a$ living on the worldvolume is

$$\mathcal{F}_{ab} = \partial_a b_b - \partial_b b_a + 1/(2\pi\alpha')\partial_a X^\Lambda \partial_b X^\Pi k^\Sigma C_{\Sigma\Lambda\Pi} \,. \tag{3.5}$$

The WZ piece is

$$S_{\mathrm{WZ}} = \frac{2\pi\alpha'}{(2\pi)^6 l_{\mathrm{P}}^7}\int K_7 \,, \tag{3.6}$$

where the K_7 is the field strength of a non-propagating worldvolume 6-form

$$\begin{aligned} K_7 = {} & d\omega_6 + \frac{1}{2\pi\alpha'}(i_k N_8) + (i_k C_6)\wedge \mathcal{F}_\in - \\ & - \frac{2\pi\alpha'}{(3!)^2} DX^\Lambda \wedge DX^\Pi \wedge DX^\Sigma C_{\Lambda\Pi\Sigma}\wedge (i_k C_3)\wedge (i_k C_3) - \\ & - \frac{1}{3!} DX^\Lambda \wedge DX^\Pi \wedge DX^\Sigma C_{\Lambda\Pi\Sigma}\wedge (i_k C_3)\wedge db - \\ & - \frac{1}{2\cdot 3!}(2\pi\alpha') DX^\Lambda \wedge DX^\Pi \wedge DX^\Sigma C_{\Lambda\Pi\Sigma}\wedge db \wedge db - \\ & - \frac{1}{3!}(2\pi\alpha')^2 A \wedge db \wedge db \wedge db \,, \end{aligned} \tag{3.7}$$

where, following [20] we wrote explicitly the pull-backs only when the covariant derivatives where involved. The 8-form N_8 is by definition the Poincaré dual of the Killing 1-form.

The action of the KKM is invariant under certain gauge transformations. We list only the ones which are relevant for this paper:

$$\begin{aligned} \delta C_3 &= d\chi_2 \,, \\ \delta i_k C_6 &= d i_k \chi_5 - \frac{1}{2} d i_k \chi_2 \wedge C_3 + \frac{1}{2} d\chi_2 \wedge i_k C_3 \,, \\ \delta i_k N_8 &= d i_k \chi_5 \wedge (i_k C_3) + \frac{1}{3} d\chi_2 \wedge i_k C_3 \wedge i_k C_3 - \frac{1}{3} d i_k \chi_2 \wedge C_3 \wedge i_k C_3 \,. \end{aligned} \tag{3.8}$$

From (3.8) we infer a gauge invariant definition of the field strength of $i_k N_8$:

$$d i_k N_8 = i_k * dk - i_k C_6 \wedge i_k F_4 - \frac{1}{6} C_3 \wedge i_k C_3 \wedge i_k F_4 - \frac{1}{6} i_k C_3 \wedge i_k C_3 \wedge F_4 \,. \tag{3.9}$$

We can convince ourselves that by varying (3.9) we get the same contributions we obtain by taking an exterior derivative in (3.8).

3.2 The KKM probe

We begin by testing the perturbed background geometry with a KKM probe. The monopole is wrapped on a 2-sphere in the (578) plane (anticipating, this will be one of the planes where polarization is most likely to occur) has 5 flat directions along $x^0, \ldots, x^4$, and has an M5 charge $n \ll N$. The potential for more general orientations of the two-sphere will be considered in the following sections. The isometry direction is $k^\Lambda = (R_{11}/l_{\mathrm{P}})\delta^{\Lambda 11}$. As discussed in the previous subsection this is a Killing vector of the perturbed $AdS_7 \times S_4$ geometry.

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As one can see from the WZ term (3.7), the monopole will have M5 charge n if we turn on its worldvolume 2-form field strength $\mathcal{F}_2$:

$$\frac{1}{(2\pi)^6 l_{\rm P}^7}\frac{R_{11}}{l_{\rm P}}\int_{S_2}(2\pi\alpha')\mathcal{F}_2 = (2\pi R_{11})T_{M5}\ n = \frac{2\pi R_{11}}{(2\pi)^5 l_{\rm P}^6}n\,, \tag{3.10}$$

which gives

$$(2\pi\alpha')\mathcal{F}_2 = (2\pi\alpha')\mathcal{F}_{\theta\phi}d\theta\wedge d\phi = \frac{1}{2}2n\pi l_{\rm P}^2\sin\theta d\theta\wedge d\phi\,. \tag{3.11}$$

We work in the regime in which the M5 contribution to the BI action of the probe monopole is dominant. Since we know the form of F_2, we can rewrite the BI action as

$$-\frac{1}{(2\pi)^6 l_{\rm P}^7}\int_{S_2}d\theta d\phi\ k^2\sqrt{\det G_{||}}\sqrt{\det G_{ab}+\det\left(\frac{(2\pi\alpha')\mathcal{F}_{ab}}{k}\right)}, \tag{3.12}$$

where $G_{||}$ denotes the induced metric on the 5 space-time directions common to the KKM and the M5-brane, and a, b are coordinates on the 2-sphere. We have assumed the integration over $dx^0\cdots dx^4$ to be implicit, and we will assume this throughout this section. Using the induced metric $G_{ab} = \{G_{\theta\theta} = Z^{2/3}r^2, G_{\phi\phi} = Z^{2/3}r^2\sin^2\theta\}$, we find the condition that the M5 contribution to the BI action dominates to be

$$\det\left(\frac{(2\pi\alpha')\mathcal{F}_{ab}}{k}\right) = \frac{(2\pi\alpha')^2}{2k^2}\det(G_{ab})\mathcal{F}_{ab}\mathcal{F}^{ab} \gg \det G_{ab}\,, \tag{3.13}$$

or

$$\frac{(2\pi)^2 l_{\rm P}^6 n^2 Z^{-4/3}r^{-4}Z^{1/3}}{4R_{11}^2} \gg 1\,.^5 \tag{3.14}$$

Under this assumption we can Taylor expand the BI action

$$-\frac{1}{(2\pi)^6 l_{\rm P}^7}\int_{S_2}d\theta d\phi k^2\sqrt{\det G_{||}}\sqrt{\det\left(\frac{(2\pi\alpha')\mathcal{F}_{ab}}{k}\right)}\left[1+\frac{k^2\det G_{ab}}{2(2\pi\alpha')^2\det\mathcal{F}_{ab}}\right] \tag{3.15}$$

and find the leading contribution

$$-\frac{R_{11}}{2(2\pi)^5 l_{\rm P}^6}\int_{S_2}d\theta d\phi Z^{-1}n\sin\theta = -\frac{R_{11}}{(2\pi)^4 l_{\rm P}^6}Z^{-1}n \tag{3.16}$$

and the subleading one

$$-\frac{R_{11}^3}{(2\pi)^7 l_{\rm P}^{12}}\int_{S_2}d\theta d\phi Z^{-1/2}Z^{-5/6}\frac{Z^{4/3}r^4\sin\theta}{n} = -\frac{2R_{11}^3}{(2\pi)^6 l_{\rm P}^{12}}\frac{r^4}{n}\,. \tag{3.17}$$

[5] Note this condition is not independent of r as one might expect in a conformal theory. The source of non conformality is the dependence of the effective size of the 11'th dimension (which is also the KKM mass) on r.

The leading terms in the BI and WZ actions represent M5-M5 interaction, and cancel as expected. More explicitly, the WZ leading contribution is

$$\frac{1}{(2\pi)^6 l_{\rm P}^7}\int i_k C_6^{\mathbf{0}}\wedge(2\pi\alpha')\mathcal{F}_2=\frac{1}{(2\pi)^5 l_{\rm P}^6}\frac{R_{11}}{2}\int_{S_2}d\theta d\phi Z^{-1}n\sin\theta\,, \tag{3.18}$$

and cancels against (3.16).

The subleading contribution to the WZ term comes from $\int i_k N_8$. When the bulk graviphoton and 3-form perturbation is turned on, several terms in (3.9) ($i_k *^{\mathbf{0}} dk^{\mathbf{1}} + i_k *^{\mathbf{1}} dk^{\mathbf{0}} + i_k C_6^{\mathbf{0}}\wedge i_k F_4^{\mathbf{1}}$) become non zero. We find

$$d i_k N_8=\frac{R_{11}^2}{l_{\rm P}^2}\left[Z^{-1}\left(*_5 d\left(H_1 Z^{1/3}\right)+G_3\right)\right]\wedge dx^0\cdots\wedge dx^4 \tag{3.19}$$

which has the solution $i_k N_8=(1/3!)(R_{11}^2/l_{\rm P}^2)T_{mnp}x^m dx^n\wedge dx^p\wedge dx^0\cdots\wedge dx^4$. It is interesting to notice that $i_k N_8$ only depends on the combination $Z^{-1}[*_5 d(H_1 Z^{1/3})+G_3)]$, which is independent of the distribution of the M5-branes, as we discussed in section 2.2.

We can see from (2.17) that the WZ subleading contribution is the largest if the 2 sphere is in the (578) or the (679) planes (depending on the signs and magnitudes of m_1 and m_2). We will assume from now on that both m_1 and m_2 are positive, and call their sum m. Thus the test brane will have a local minima in both the (578) and (679) planes when $m_2\approx 0$, and only in the (578) for $m_2\approx m_1$.

Thus, the WZ contribution to the action is

$$\frac{1}{(2\pi)^6 l_{\rm P}^7}\int i_k N_8=m\frac{4\pi R_{11}^2}{(2\pi)^6 l_{\rm P}^9}r^3\,. \tag{3.20}$$

Besides the BI and the WZ contributions found above there is yet another contribution of the same order which comes from the second order perturbation to the fields, and which will be discussed in the next chapter. The polarization radius r_0 is of the same order as the radius where $V_{\rm BI}+V_{\rm WZ}$ is minimized. Thus

$$r_0\sim\frac{nml_{\rm P}^3}{R_{11}}\,. \tag{3.21}$$

Substituting in (3.14) we find that we can treat the KKM as a probe if

$$n\gg R_{11}mN\,. \tag{3.22}$$

We can also ask if the strength of the perturbation is anywhere larger than the energy density in the unperturbed AdS background

$$\frac{|F_4{}^{\mathbf{1}}|^2}{|F_4{}^{\mathbf{0}}|^2}\sim\frac{r^{-1}Rm^2}{R^{-2}}\sim\frac{m^2 l_{\rm P}^3 N}{r}\,. \tag{3.23}$$

The fraction (3.23) grows as r decreases towards r_0. Nevertheless, inside the shell we expect this ratio to be of the same order as at r_0. By estimating (3.23) at the polarization radius we can see that the perturbation is always small if (3.22) holds.

We would like to examine more the physical implications of (3.22), which is necessary for this polarization picture to be correct. Since in the final computation n will be replaced by N, (3.22) is equivalent to $R_{11}m \ll 1$, which seems to imply that we are in fact describing the $4+1$-dimensional super Yang Mills theory living on a stack of coincident D4-branes rather than the compactified $(2,0)$ theory.

In order to understand the dimension of the theory, we briefly review the IMSY investigation ([21]-chapter 6) of the *unperturbed* system. We are in fact dealing with only one theory, which has 3 weakly coupled descriptions.

For very low energies, the IIA supergravity curvature is very large, and the weakly coupled description is the $4+1$-dimensional SYM. For higher energies (radii) IIA supergravity becomes weakly coupled. At even higher energies, the dilaton becomes very large, and the weakly coupled description of the physics is given by 11-dimensional supergravity. We should notice that the dimension of the theory jumps from $4+1$ to $5+1$ when the dimension of the "bulk" jumps from $5+1$ to $6+1$; this happens when 11-dimensional supergravity starts taking over form IIA supergravity.

In our case, one can easily estimate the effective radius of the 11'th dimension $R_{11}^{\mathrm{physical}} = R_{11}Z^{-1/6}$ and find its lowest bulk value to be $N^{1/3}(mR_{11})^{1/2}l_p$. This implies that for large N the 10-dimensional string coupling constant is large everywhere in the bulk, and thus 11-dimensional supergravity is the appropriate description. Thus, we are working in the regime where "the" theory[6] has $5+1$-dimensional character and 11-dimensional sugra in the bulk with polarized M5-branes gives its weakly coupled description.

Unlike the original $(2,0)$ theory on a circle which flows in the IR to a free $4+1$-dimensional theory ($4+1$ super Yang-Mills), our perturbed $(2,0)$ theory flows in the IR to a $5+1$-dimensional theory.

Thus, $R_{11}m \ll 1$ implies only that the perturbation we are turning on has the energy scale in the range where $4+1$ SYM was valid. However, now there is no more $4+1$ SYM. We perturbed the conformal $(2,0)$ theory by a relevant deformation which leaves UV physics unchanged but makes the RG flow never change dimension. Thus, one can say that the $4+1$ SYM regime is effectively excised from the RG trajectory.

The fact that we can only have a polarization state in the regime $R_{11}m \ll 1$ seems puzzling if the theory we are dealing with is supersymmetric. Indeed, one does not expect a vacuum state to just go away, as we increase adiabatically m or R_{11}. Nevertheless, this is exactly what happens. The solution to this puzzle will be found in the next chapter — in order to polarize the branes one needs to break all supersymmetry.

[6] Whether one might call it $(2,0)$ or $4+1$ SYM is a matter of terminology.

3.3 The full problem

We use the results of the previous section to solve the full problem of a self-interacting KKM with large M5 charge. The self-interaction potential can be found by bringing from infinity probe KKM's with M5 charge in the background of polarized M5 branes. As we will immediately show, the potential these probes feel is unaffected by the distribution of the M5-branes. Therefore, the total self-interaction potential will be the same as the potential of a single shell, with n replaced by N.

The geometry of a distribution of M5 will still be given by (2.4), but the warp factor Z is a superposition of harmonic functions. For the case of N- M5-branes uniformly distributed on a 2-sphere of radius r_0 in the (578) hyperplane, the warp factor is

$$\begin{aligned} Z &= \frac{1}{2}\int_0^\pi d\theta \frac{R^3 \sin\theta}{(r_0^2+r_1^2+r_2^2-2r_0 r_1\cos\theta)^{3/2}} \\ &= \frac{R^3}{2r_0 r_1}\left(\frac{1}{\sqrt{(r_0-r_1)^2+r_2^2}}-\frac{1}{\sqrt{(r_0+r_1)^2+r_2^2}}\right), \end{aligned} \tag{3.24}$$

where r_1 is the radius in the (578) hyperplane and r_2 is the radius in the (69) plane. For large r_1, r_2 we recover the AdS_7 warp factor. If the M5-branes are distributed on several shells, then the total warp factor will be the sum of each shell Z factor.

As we explained in section 2, eqs. (2.11) and (2.13) imply that $Z^{-1}[*_5 d(hZ^{1/3})+G_3)]$ is a harmonic function, and therefore it is given by its value at infinity, regardless of the value of Z. This is exactly the combination that enters the WZ term of the KKM. Also, in the large-N limit, the BI term can be expanded as in (3.15), and the cancellation of the leading WZ and BI takes place as before. The subleading terms are unchanged, and have no Z dependence. We thus conclude, that for each M5 shell the potential is the same as the probe potential.

The most general polarized configuration consistent with the symmetries is a KKM wrapped on a two-ellipsoid:

$$x_7 = \underline{x_7}\cos\alpha\,, \qquad z_1 = \underline{z_1}\sin\alpha\cos\beta\,, \qquad z_2 = \underline{z_2}\sin\alpha\sin\beta\,, \tag{3.25}$$

where z_1 and z_2 are defined in (2.14), α and β parametrize the embedding, and $\underline{x_7}, \underline{z_1}$ and $\underline{z_2}$ give the length and orientation the semiaxes. It is a straightforward exercise to express the contribution of the BI and WZ potentials in terms of the semiaxes of the probe ellipsoid

$$\begin{aligned} V_{\rm BI+WZ} =& \frac{2R_{11}^3}{3n(2\pi)^6 l_{\rm P}^{12}}\left(|z_1|^2x_7^2+|z_2|^2x_7^2+|z_1|^2|z_2|^2\right) - \\ &-\frac{4\pi R_{11}^2}{(2\pi)^6 l_{\rm P}^9}\,{\rm Re}(m_1 z_1\bar{z}_2 x_7 + m_2 z_1 z_2 x_7)\,, \end{aligned} \tag{3.26}$$

where we have dropped the subscripts on the semiaxes to keep notation simple. From now and throughout this chapter we will denote the semiaxes by x_7, z_1 and z_2 without an underline.

There is another term in this potential which comes from a second order correction to the background. This term has the same relevance as the first two, and is hard to compute explicitly (it has been done in [8]). We have already discussed that by giving masses to either of the two fermions we break half of the original amount of supersymmetry ((2, 0) in 6 dimensions). Thus, when one of the fermion masses (m_2 in this case) is zero, we can use supersymmetry to complete the squares in (3.26). This gives the complete effective potential

$$V_{\text{total}} = \frac{2R_{11}^3}{3n(2\pi)^6 l_{\text{P}}^{12}} \left[\left(|z_1|^2 x_7^2 + |z_2|^2 x_7^2 + |z_1|^2|z_2|^2\right) - \frac{6\pi n l_{\text{P}}^3}{R_{11}} \text{Re}(m_1 z_1 \bar{z}_2 x_7) + \right.$$
$$\left. + \left(\frac{3\pi n l_{\text{P}}^3}{2R_{11}}\right)^2 m_1^2 \left(|z_1|^2 + |z_2|^2\right) \right], \tag{3.27}$$

is derived in the usual way from the superpotential W

$$W = \sqrt{\frac{2R_{11}^3}{3n(2\pi)^6 l_{\text{P}}^{12}}} \left[z_1 z_2 x_7 - \frac{3n\pi l_{\text{P}}^3}{4R_{11}} m_1 \left(z_1^2 + z_2^2\right) \right]. \tag{3.28}$$

The potential (3.27) has only one minimum at $x_7 = z_1 = z_2 = 0$. We conclude that the polarization of M5-branes into a wrapped KKM is not possible if we have any supersymmetry left.

We remember that in [1], one had to give mass to at least 3 of the 4 complex Weyl fermions in order for polarization to occur. In our case (as well as in the D4-D6 case), we only have two fermions, so one can either give mass to one (1/2 of the fermions) which preserves $\mathcal{N} = 1$ supersymmetry but is not enough for polarization, or give mass to both fermions, break supersymmetry completely, and polarize the branes.[7]

There are two ways one can bypass this problem.[8] The first one is to consider an "almost supersymmetric" situation, when the susy breaking mass m_2 is far smaller than m_1. One can still use supersymmetry to complete the squares in (3.26), and find the potential to be

$$V = \frac{2R_{11}^3}{3n(2\pi)^6 l_{\text{P}}^{12}} \left[\left(|z_1|^2 x_7^2 + |z_2|^2 x_7^2 + |z_1|^2|z_2|^2\right) - \right.$$
$$- \frac{6\pi n l_{\text{P}}^3}{R_{11}} \text{Re}(m_1 z_1 \bar{z}_2 x_7 + m_2 z_1 z_2 x_7) +$$
$$\left. + \left(\frac{3n l_{\text{P}}^3}{2R_{11}}\right)^2 \left(m_1^2 |z_1|^2 + m_1^2 |z_2|^2 + 4m_2^2 x_7^2\right) \right]. \tag{3.29}$$

[7]Naively one might think that by T-dualizing the D3-D5 polarization of [1] to a D4-D6 configuration we do not lose supersymmetry. We can see that this is not the case, both by investigating the theory on the brane (one needs to pair the D3 fermions two by two to obtain D4-brane fermions, and thus cannot consistently give mass to one fermion and a half), or by realizing that the bulk in [1] has no isometry along the T-duality direction.

[8]Similar methods and a more thorough discussion can be found in [5].

In this case, the M5-branes will polarize into a monopole wrapped on an ellipsoid. The new potential has a minimum at

$$z_1 = z_2 = \sqrt{\frac{m_1 m_2}{2}} \frac{3\pi n l_{\mathrm{P}}^3}{R_{11}} = x_5 = x_8 \,, \qquad x_7 = \frac{3\pi n l_{\mathrm{P}}^3}{2R_{11}} m_1 \,. \tag{3.30}$$

Clearly, the ellipsoid is very elongated in the x_7 direction, and in the supersymmetric case it degenerates into a line.

The second way to find the last term of the potential (3.26) is to consider an SO(3) invariant non-supersymmetric configuration. The last term of (3.27) represents a mass given to 4 of the 5 scalars in our theory. The most general scalar mass term can be written as:

$$m^2 \left(\Phi_5^2 + \Phi_6^2 + \Phi_8^2 + \Phi_9^2 + \Phi_7^2\right) + \mu_{ij} \Phi_i \Phi_j \,, \tag{3.31}$$

where the first piece is an $L = 0$ combination and the second one is a combination of various $L = 2$ modes. In the supersymmetric case Φ_7 is part of the same multiplet as the self-dual 3-form and half of the fermions, and remains massless, while the other 4 scalars receive identical masses. Thus, supersymmetry constraints the last term of (3.31) to be

$$\mu_{ij} \Phi_i \Phi_j = \frac{m^2}{4} \left(\Phi_5^2 + \Phi_6^2 + \Phi_8^2 + \Phi_9^2 - 4\Phi_7^2\right) . \tag{3.32}$$

In the non-supersymmetric case only the $L = 0$ mode is determined by the back reaction of the first order fields. The $L = 2$ modes can be specified independently in the boundary theory (this issue is discussed very briefly here, for a more thorough treatment we refer the reader to [1, 5, 6, 9]). Let us consider the case when both fermions have equal masses $m_1 = m_2 = m$. We can turn off (3.32) to restore the SO(5) symmetry between x_5, x_6, x_8, x_9 and x_7, and turn on an $L = 2$ mode which preserves the SO(3) symmetry in the polarization plane spanned by x_5, x_8, x_7.

$$-\Lambda \left(\Phi_5^2 + \Phi_8^2 + \Phi_7^2 - \frac{3}{2}\Phi_6^2 - \frac{3}{2}\Phi_9^2\right) . \tag{3.33}$$

For Λ of order m^2 or higher, a state in which the KKM is wrapped on a sphere of radius of order r_0 will have energy less than the vacuum energy. Thus, the M5-branes will polarize into a KKM wrapped on a 2-sphere of radius of order $r_0 \sim (nml_{\mathrm{P}}^3)/R_{11}$, in the 578 hyperplane.

When there is more than one shell, the potential is the sum of individual pieces of the form (3.26). One can also estimate the polarization potential for Q coincident KKM shells and find that $r_{\mathrm{polarization}} \sim r_0/Q$. This configuration should also be part of the many vacua the perturbed theory should have.

Thus, the vacua of the theory will consist of multiple M5-KKM (ellipsoidal or spherical) shells. We will concentrate in the rest of this paper on the SO(3) invariant non-supersymmetric theories which correspond to polarization into a 2-sphere. The generalization to the almost supersymmetric theories can be easily done.

3.4 The near shell solution

The KK monopole solution without M5 charge, is given by the $R^7\times$ Taub-NUT space,

$$\begin{aligned} ds^2 &= \eta_{\mu\nu}dx^\mu dx^\nu + V(r)\left(dx^{11}+4m(1-\cos\theta)d\phi\right)^2 + \\ &\quad + V(r)^{-1}\left(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right), \\ V(r)^{-1} &= 1+\frac{4m}{r}. \end{aligned} \tag{3.34}$$

Near $r=0$ the metric $ds^2 \sim (dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2+r^2(dx^{11}/4m)^2)$ has a nut singularity which is removed only if x^{11} has period $16\pi m = 2\pi R_{11}$. Thus the charge of the monopole is proportional to the length of the periodic isometry coordinate. This explains why the effective tension of the monopole scales with R_{11}^2. The requirement for the periodicity of x^{11} can also be extracted from the requirement that there is no Dirac string singularity (this is explained in more detail in [22]).

One might be worried that upon adding a very large M5 charge to the monopole, the metric would be changed and the condition for not having a Dirac string would change from $R_{11}\sim m$, to some formula which also includes the M5 charge. We thus need to examine the metric of a flat KKM with large M5 charge. This metric can be easily found [23] by lifting the D4-D6 metric [24] obtained by T-dualizing smeared tilted D5 branes. If we call α the original D5 tilting angle, and $\tilde{N}$ the original number of D5-branes, we obtain the flat KKM/M5 metric to be:

$$\begin{aligned} ds^2 &= h^{-1/3}dx_{\|}^2 + h^{2/3}\left(dx_6^2+dx_9^2\right) + Zh^{-1/3}\left(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right) + \\ &\quad + h^{2/3}Z^{-1}\left(dx^{11}+\tilde{N}\cos\alpha(1-\cos\theta)d\phi\right)^2, \end{aligned} \tag{3.35}$$

where

$$\begin{aligned} Z &= 1+\frac{\tilde{N}l_{\mathrm{P}}}{r}, \\ h &= \frac{1+\tilde{N}l_{\mathrm{P}}/r}{1+\tilde{N}l_{\mathrm{P}}\cos^2\alpha/r}. \end{aligned} \tag{3.36}$$

We can see that when $\cos\alpha = 0$, h and Z are equal and we recover the metric of multiple M5-branes smeared on a 2-plane. When $\cos\alpha = 1, h = 1$ and we recover the plain KKM solution (3.34). By comparing (3.35) with (3.24) near the shell we can identify the coordinates r and $\rho = \sqrt{(r_0-r_1)^2+r_2^2}$. We can also express the dummy coefficients $\tilde{N}$ and α in (3.36) in terms of our physical near shell parameters by comparing (3.35) with (3.24) and (3.34). We find the near horizon limits of Z and h:

$$Z = \frac{\pi l_p^3 N}{2r_0^2\rho}, \qquad h = \frac{\pi l_p^3 N}{2r_0^2\left(\rho+\frac{R_{11}^2 r_0^2}{2\pi l_p^3 N}\right)}. \tag{3.37}$$

We can also express the factor $\tilde{N}\cos\alpha$ from (3.35) as:

$$\tilde{N} l_{\rm P} \cos\alpha = \frac{R_{11}}{2} \,. \tag{3.38}$$

Thus, dx^{11} and $d\phi$ appear with the same relative coefficient as in (3.34) and therefore the condition for no Dirac string is unaffected by the presence of M5 charge.

The KKM contribution to the metric becomes important only very close to the shell, at $\rho \sim R_{11}^2 r_0^2/2\pi l_{\rm P}^3 N \sim N m^2 l_{\rm P}^3 \equiv \rho_{\rm crossover}$. We can see from (3.21) that $\rho_{\rm crossover}/r_0 \sim m R_{11} \ll 1$, and thus the picture of the KKM wrapped on a 2-sphere is consistent.

We can find the range of validity of the supergravity approximation by comparing $l_{\rm P}$ to the radius of the transverse sphere in (3.35). We find supergravity valid if

$$\left(\frac{\rho}{\rho_{\rm crossover}}\right)^{3/2} N m^2 R_{11}^2 \gg 1 \,, \tag{3.39}$$

which is easily satisfied at large N, even for $\rho \ll \rho_{\rm crossover}$. When ρ is so small that (3.39) is not satisfied, the physics starts being described by the gauge theory living on the KKM.

4. The properties of the vacua of the perturbed theory

We are now at the stage to begin exploring the properties of the M5-KKM vacua we found above. We know that the original $(2,0)$ has tensionless little strings, and "baryon vertices". We expect to find these objects in the perturbed theory as well. Moreover, since now we have many vacua we expect to have domain walls separating them. We would like to find the bulk M-theory configurations corresponding to these objects.

4.1 Little strings

The non-perturbed $(2,0)$ theory has two types of self-dual (little) strings (the string boundary of an M2-brane on an M5 brane is naturally charged under the self-dual 3-form field strength on the M5 worldvolume). The "quark" little strings are massive, and correspond to M2-branes lowered from infinity and ending on the stack of M5-branes. The other little strings are M2-branes going from one M5-brane to another, and are massless when the M5-branes are coincident.

The interaction energy between two little strings can be found by estimating the energy of the bulk M2-brane whose boundary the strings are [25]. In the UV, the M2-brane stays very near the AdS boundary, and thus does not know that the M5-branes sourcing the geometry are polarized. This is expected — giving a small mass to massless fields should not affect UV physics. In the infrared, the M2-brane gets near the M5-KKM shell and interacts with it, and thus the physics will change.

Let us consider two largely separated parallel strings which are the boundary of an M2-brane which comes very near the KKM-M5 shell. In the limit when the strings are very far apart, the largest contribution to the interaction energy comes from the part of the M2-brane which is parallel to the shell.

Since we have broken Lorentz invariance by turning on an R-current, the interaction energy will depend on the orientation as well as on the direction of separation of the little strings.

For the sake of clarity, let us explain in a bit more detail what the setup is. The polarized M5's span the $0, 1, 2, 3, 4, 11$ directions, with the 11th direction compactified on a circle of radius R_{11}. The KKM in which the M5's polarize has the topology $R^4 \times S_2$, with the 11th direction being the isometry direction. If we use polar coordinates in the plane of the S_2, the KKM is aligned along $0, 1, 2, 3, 4, \theta, \phi$. The geometry at infinity is still $AdS_7 \times S_4$, with the boundary of AdS_7 spanned by $0, 1, 2, 3, 4, 11$. The radial coordinate of AdS_7 is r.

We distinguish two possible situations, depending on the orientation and separation of the little strings:

i) None of the directions of the M2-brane is 11. This corresponds to non-11 strings separated along any direction but 11. The action of the M2-brane is proportional to the square root of the determinant of the induced metric. From (3.35) we find this to be $\sqrt{\det(G_{\text{induced}})} = h^{-1/2}$. Since h remains finite as we approach the shell ($r \to 0$), the interaction energy of two little strings is proportional to their separation. Thus the strings are confined.

One can easily estimate the tension of the confining "flux tube", However, there is a caveat — the supergravity approximation is valid at $\rho \sim \rho_{\text{crossover}}$ only for $Nm^2 R_{11}$ large enough (3.39). Unless this condition is satisfied, we cannot use (3.35) at $\rho \approx \rho_{\text{crossover}}$ to reliably compute this tension. Thus,

$$T = T_{M2} h^{-1/2}|_{\min} \sim \frac{r_0^2 R_{11}}{l_p^6 N^2} \sim \frac{m^2 N}{R_{11}} \,. \tag{4.1}$$

ii) One of the direction of the M2-brane is 11. This corresponds to either non-11 strings separated along the 11'th directions, or to (0,11) strings separated along any other direction. In this case $\det(G_{\text{induced}}) = Z^{-1/2}$. Since Z^{-1} goes to zero as $\rho \to 0$, it does not cost any energy to take the strings further apart. Thus the strings are "screened". The M-theory explanation of this phenomenon is that the 2 sides of the M2-brane attach to the shell, and they are free to move apart.

We also expect the tensionless strings to get an effective tension as we turn on m. We can use a naive argument to estimate this tension. The original tensionless strings come from M2-branes stretched between coincident M5-branes. When the N M5 branes are spread on a 2-sphere of radius r_0, the average separation between them will be of order $r_0 N^{-1/2}$, which will give an effective tension of order

$$T_{\text{effective}} \sim r_0 N^{-1/2} (G_{55} G_{00} G_{ii})^{1/2} = r_0 N^{-1/2} \tag{4.2}$$

for $(0, i)$ little strings, and

$$T_{\text{effective}} \sim r_0 N^{-1/2} (G_{55} G_{00} G_{11,11})^{1/2} = r_0 N^{-1/2} h Z^{-1} \tag{4.3}$$

for $(0, 11)$ little strings. It appears that the $(0, 11)$ strings remain massless. However, more is happening. At the KKM core, the size of the 11'th dimension is zero, and thus a string originally winding on the 11'th direction disappears from the spectrum.

4.2 Domain walls

The perturbed theory has many isolated vacua, which must be separated by domain walls. If the two vacua have the same number of KKM's but different distributions of M5 charge, the KKM's will generically intersect on a 2 sphere on which they will exchange M5 charge. This bulk configuration is dual to the boundary theory domain wall. The tension of the domain wall is given by the bending tension of the configuration.

If the vacua have different numbers of KKM's, $Q_1 \neq Q_2$, the monopoles intersect again on a 2-sphere where they exchange M5 charge. By KKM charge conservation, a KKM with charge $Q_2 - Q_1$ will fill the 3-ball whose boundary the 2-sphere is. The tension will have an extra piece coming from the energy of the KKM's which fill the 2-sphere.

If we do not have any supersymmetry, the two vacua separated by the domain wall will have generically different energies, and thus the domain wall will be moving. In the limit in which supersymmetry is restored, the energies of the vacua are almost the same, and the domain walls will be almost stationary.

4.3 Baryon vertices

In the unperturbed $AdS_7 \times S^4$ background the string baryon vertex is an M5-brane wrapped on S_4, with N M2-branes lowered from the boundary ending on it. This configuration has been discussed thoroughly in [26, 27].

Let us analyze what happens to the baryon vertex in the background of polarized M5's. In the UV the physics is not affected by the perturbation, and thus the baryon vertex corresponds to an M5-brane wrapped on S_4. As we flow to the IR, the wrapped M5 eventually reaches the KKM shell and crosses it. In order to see if any objects are created via the Hanany-Witten effect [28], let us "zoom in" on a region near the shell where the branes look almost flat and $\hat{r} \parallel \hat{x}_5$.

Since the 11'th direction was made special by turning on an R-current, we will first analyze a baryon vertex which is not extended along the 11'th direction, and then analyze one which is. Let us consider a baryon vertex M5-brane locally extended along the $0, 1, 6, 7, 8, 9$ directions, and a KKM locally extended along $0, 1, 2, 3, 4, 7, 8$. We claim that when we bring the M5-brane along x_5 and cross the KKM another M5-brane is created, extended in the $0, 1, 5, 7, 8, 11$ directions. It appears puzzling that the new brane is extended along the 11'th direction. Nevertheless, we believe that this is indeed the case. There are three arguments one can give to support this. The first is that one obtains this configuration by lifting to 11 dimensions the Hanany-Witten effect at the crossing of an NS5-brane and a D6-brane, when a D4 brane is created transverse to the other two. The second is that, the KKM end of the created brane is magnetic with respect to the 2-form field strength on the monopole, as it should be. The third one will follow when we investigate the properties of the new baryon vertex.

As one brings the wrapped M5 through the KKM shell, the M2-branes hanging from the boundary will end on the intersection of the created M5 with the KKM, and there will be nothing preventing the original wrapped M5 to shrink to zero size.

Thus, the bulk dual of the new baryon vertex is an M5-brane which fills the 3-ball whose boundary the KKM 2-sphere is, and which has one direction along the isometry direction. To justify the baryon vertex name, we look at the KKM action (3.7) term

$$\int d^7\xi Dx^\Lambda \wedge Dx^\Pi \wedge Dx^\Sigma \wedge C_{\Lambda\Pi\Sigma} \wedge db_1 \wedge db_1 \,, \tag{4.4}$$

where b_1 is the 1-form on the worldvolume of the monopole. Let us denote db_1 by F_2. Due to the magnetic ending of the created M5, the Bianchi identity gets modified and this term will no longer be gauge invariant under the background gauge transformation $\delta C_3 = d\chi_2$:

$$\begin{aligned} \delta \int F_2(\text{monopole}) \wedge F_2(\text{dissolved}) \wedge C_3 = \\ = - \int dF_2(\text{monopole}) \wedge F_2(\text{dissolved}) \wedge \chi_2 \end{aligned} \tag{4.5}$$

unless N M2's descend from the boundary to the M5-KKM junction.

One might also be worried that the new baryon vertex is extended along three directions $(0, 1, 11)$, and might have a higher energy than the original baryon vertex which is only extended along $(0, 1)$. We can calculate this energy using the induced metric (3.24)

$$\begin{aligned} E &= \frac{1}{l_{\rm P}^6} R_{11} 4\pi \int dx^0 dx^1 dr_1 r_1^2 Z^{1/2} \\ &= \frac{4\pi R_{11}}{l_{\rm P}^6} \int dx^0 dx^1 \int_0^{r_0} dr_1 r_1^2 \sqrt{\frac{R^3}{2r_0 r_1}\left(\frac{1}{r_0 - r_1} - \frac{1}{r_0 + r_1}\right)} \\ &= \int dx^0 dx^1 \frac{\sqrt{2}\pi^2 R_{11} (R r_0)^{3/2}}{l_{\rm P}^6} \,. \end{aligned} \tag{4.6}$$

Substituting r_0 we obtain the new baryon vertex energy:

$$E \sim \int dx^0 dx^1 \sqrt{N} m^{3/2} R_{11}^{-1/2} \,. \tag{4.7}$$

The energy of an M5-brane wrapped on S^4 of radius $\rho_{\text{crossover}}$, namely the old baryon vertex at the point where supergravity stops being valid (and M-theory phenomena like the Hanany-Witten effect described above appear), is

$$E \sim T_{M5} \int dx^0 dx^1 Z(\rho) \rho^4_{\text{crossover}} \sim \int dx^0 dx^1 N^2 m^4 R^2_{11}\,, \tag{4.8}$$

where we used the near shell metric to compute the induced metric on S^4.

Thus the condition that the new configuration has a lower energy is

$$1 \ll N^{3/2}(mR_{11})^{5/2} \tag{4.9}$$

which is easily satisfied for sufficiently large N.

When the UV baryon vertex is extended along x_{11}, there does not seem to be any object created when it passes through the KKM shell. This is consistent with the fact that strings extended along x_{11} are screened. Once the baryon vertex gets near the KKM shell, these strings can attach to it, and there is nothing to prevent the M5-brane from collapsing. Thus the baryon vertex extended along x_{11} disappears in the IR.

5. Conclusions and future directions

We investigated the effect of a fermion R-current perturbation on the $(2,0)$ theory living on the worldvolume of N coincident M5-branes. We found the dual M-theory background to contain the M5-branes polarized into Kaluza-Klein monopoles, with the isometry direction of the monopoles on the worldvolume of the M5-branes. However, we showed that polarization is possible only when all the supersymmetry is broken.

We examined the properties of the vacua of the perturbed theory. We discovered that the $(0,i)$ tensionless little strings acquire a tension, while the $(0,11)$ strings disappear from the spectrum.

The "quark" little strings extended along the isometry directions are screened, and the strings which are not extended along x^{11} are confined. We calculated the tension of the confining flux tube.

We also found the new IR description of the string baryon vertex, by flowing the UV baryon vertex (which is an M5-brane wrapped on S_4) to the IR. A new M5-brane is formed when the UV vertex crosses the M5-KKM shell. This new baryon vertex fills the 3-ball whose boundary the KKM is, and is extended along the isometry direction. The remaining confining little strings will have precisely this baryon vertex. We checked that the new baryon configuration is energetically preferred to the old one. We have also given an M-theory description to the domain walls in the perturbed theory.

This paper completes a missing piece in the understanding of brane polarization, by showing that M5-branes can and do polarize into Kaluza-Klein monopoles. The new configuration might be a good setup for an entropy computation in the spirit of [8] to see if the correction to the entropy has N^3 behavior.

It is also interesting to ask if the M5 $\rightarrow$ KKM polarization exhausts the series of possible brane polarizations. Besides the Dp $\rightarrow$ D(p+2), the D2,D3 $\rightarrow$ NS5 polarizations explored in [1, 5], and the F1 $\rightarrow$ D4 polarization which is the reduction of the M2 $\rightarrow$ M5 polarization studied in [4], it looks like type-IIA F1 strings and D4-branes might polarize into D2-branes and, respectively, NS5-branes, wrapped on a one-sphere. This is supported by the fact that a D2/NS5-brane wrapped on a one-sphere can have quantized F1/D4 charge. These possible polarizations would be however different from the ones studied so far, for two reasons.

First, the F1 string and the D2-brane come from the same object in M-theory. Lifting a D2 wrapped on a one sphere with a large F1 charge to 11 dimensions one obtains an M2-brane wrapped in a complicated way around the 11'th direction. Even if this configuration might be stable in a perturbed theory, it is quite unlikely that it will be an isolated vacuum like before.

Second, one can also find the powers of r which appear in the polarization potential (the equivalent of (3.27)).[9] Since only two of the scalar fields become non commutative for this polarization, the superpotential will contain only terms with two fields, and thus the potential will contain three r^2 terms. Thus, in the supersymmetric case this potential will either vanish (in which case we expect to have a moduli space of possible polarization vacua) or have only one minimum at $r = 0$ (which means no polarization). Before this paper was published [29] appeared in which this investigation was carried out, and the moduli space of polarization vacua suggested above was indeed found.

Acknowledgments

We would like to acknowledge useful conversations with Joe Polchinski, Aleksey Nudelman, Alex Buchel, Wati Taylor, and Radu Roiban. We would also like to thank Mukund Rangamani for bringing [11] to our attention. The work of I. B. was supported in part by NSF grant PHY97-22022 and the work of D. V. by NSF grant 9722101.

A. A few useful formulas

We list several properties of the antisymmetric 2- and 3-tensors which form a basis for forms on the transverse space.

[9] We thank W. Taylor for bringing this to our attention.

$$
\begin{aligned}
V_3 &= \frac{1}{3!}\left(\frac{x^q x^m}{r^2}T_{qnp} + 2 \text{ more}\right) dx^m \wedge dx^n \wedge dx^p\,, \\
V_2 &= \frac{1}{2!}\left(\frac{x^q x^i}{r^2}T_{qj} + 1 \text{ more}\right) dx^i \wedge dx^j\,, \\
T_2 - V_2 &= *_5 V_3\,, \\
d(\ln r)\wedge V_3 &= 0\,, \\
d(\ln r)\wedge *_5 V_3 &= d(\ln r)\wedge *_5 T_3\,, \\
d(V_3) &= -3 d(\ln r)\wedge T_3\,, \\
d(*_5 V_3) &= 2 d(\ln r)\wedge *_5 T_3\,.
\end{aligned} \tag{A.1}
$$

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